Inverse quadratic interpolation method

1. Use three steps of the inverse quadratic interpolation method to approximate a root of the function $f(x) \overset{\text{def}}{=} \frac{\sin(x)}{x} + e^{-x}$ starting with $x_0 = 3.0$. $x_1 = 3.2$ and $x_2 = 3.4$, but when discarding one of the previous results, always ensure that the root continues to be bracketed.

Answer: To ten significant digits, we have [3.0, 3.2, 3.4], [3.2, **3.267199145**, 3.4],

[3.2, **3.266498284**, 3.267199145], [3.266498284, **3.266500437**, 3.267199145]

2. Use three steps of the inverse quadratic interpolation method to approximate a root of the function $f(x) \overset{\text{def}}{=} x^3 - 3x + 1$ starting with $x_0 = 1.4$, $x_1 = 1.5$ and $x_2 = 1.6$, but when discarding one of the previous results, always ensure that the root continues to be bracketed.

Answer: To ten significant digits, we have [1.4, 1.5, 1.6], [1.5, **1.532868936**, 1.6], [1.5, **1.532085101**, 1.532868936], [1.532085101, **1.532088886**, 1.532868936]

3. Consider finding the next approximation with (–0.1, –0.1), (0.1, 0.1), (0.2, 0.5). Does the next iteration of Muller's method and the inverse quadratic interpolation method produce the same results?

Answer: No. The quadratic that interpolates (–0.1, –0.1), (0.1, 0.1), (0.2, 0.5) is $10x^2 + x - 0.1$ which has a root close to $1/10\phi$, while the quadratic that interpolates the points (–0.1, –0.1), (0.1, 0.1), (0.5, 0.2) is $-1.25x^2 + x + 0.0125$ which has a constant coefficient 0.0125.